

**Dependent Samples  $t$  test**  
Chapter 11

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**Class Outline – 7-28-08**

- Return papers
  - Questions?
  - Review late policy
- Review Independent Samples  $t$ -test
- $t$  test for Related Samples – Chap. 11
- Estimation – Chap. 12
- Problem Set #4
- Questions?

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**Chap. 11 –  $t$  test for related samples**

- In the last class we discussed the independent-measures  $t$  test
- Two separate samples are compared to determine if there is a significant difference between the two
- Today we will be using a repeated-measures  $t$  test

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## Repeated $t$

- **Repeated-measures (or dependent-measures) design**— a single sample of individuals is measured more than once on the same dependent variable. The same subjects are used in all of the treatment conditions (within-subjects study)
- **Matched-subjects design** – each individual in one sample is matched with an individual in the other sample (so not the same individual, as above, but still similar)
- Because the same (or matched) subjects are being used in all treatment conditions, the subjects will not be systematically different from one treatment to another (reduces error)

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## Repeated $t$

- The  $t$  statistic for related samples is basically the same as the single-sample  $t$  statistic. What is different is that we use difference scores (the difference between the dependent variable measurements), while with the single-sample  $t$  statistic we used raw scores ( $X$  values)
- We typically get the difference score by subtracting the first score from the second score

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**Table 11-1**  
Reaction time measurements taken before and after taking an over-the-counter cold medication.

Person	Before medication ( $X_1$ )	After medication ( $X_2$ )	Difference $D$
A	215	210	-5
B	221	242	21
C	196	219	23
D	203	228	25

$$\Sigma D = 64$$

$$M_D = \frac{\Sigma D}{n} = \frac{64}{4} = 16$$

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## Repeated $t$

- Difference score:  $D = X_2 - X_1$
- If a subject has a higher score on  $X_1$  than they do on  $X_2$ , then the difference score will be negative
- The sample of difference scores ( $D$  values) will serve as the sample data for the hypothesis test (so the mean, SS and  $n$  of  $D$  scores, not the raw scores)

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## Repeated $t$ Hypotheses

- We are interested in the mean for the population of difference scores (which we identify as  $\mu_D$ )
  - Is there any difference between the two treatment conditions (if every individual was measured twice) for the general population?
- Null hypothesis:  $H_0: \mu_D = 0$
- Alternative hypothesis:  $H_1: \mu_D \neq 0$ 
  - In other words, there is a consistent, predictable difference between the two treatments

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### Figure 11-1

A sample of  $n = 4$  people is selected from the population. **Each individual is measured twice**, once in treatment I and once in treatment II, and a difference score,  $D$ , is computed for each individual. This sample of difference scores is intended to represent the population. **Note that we are using a sample of difference scores to represent a population of difference scores.** Specifically, we are interested in the mean difference for the general population. The null hypothesis states that for the general population there is no consistent or systematic difference between the two treatments, so the population mean difference is  $\mu_D = 0$ .

Subject	I	II	$D$
A	10	14	4
B	15	13	-2
C	12	15	3
D	11	12	1

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## The $t$ formula

- Remember back to single sample:

- $$t = \frac{M - \mu}{s_M}$$

- $\mu$  is calculated from null hypothesis

- Now, for repeated measures:

- $$t = \frac{M_D - \mu_D}{s_{M_D}}$$

- $\mu_D$  is calculated from null hypothesis

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## The $t$ formula

- $t$  formula for repeated-measures design

- $$t = \frac{M_D - \mu_D}{s_{M_D}}$$

- $M_D = \Sigma D / n$
- Note: there is only one  $D$  score for each subject
- A sample of  $n$  subjects will have  $n$   $D$  scores and the  $t$  statistic will have  $df = n - 1$

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## The $t$ formula

- Variance and estimated standard error computed same way as before (but now using the  $D$  scores)
- $SS = \Sigma D^2 - (\Sigma D)^2 / n$

- $$s^2 = \frac{SS}{df}$$

- $$s_{M_D} = \sqrt{\frac{s^2}{n}}$$

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## The *t* formula

- Once you've calculated the estimated standard error of the mean difference, you can plug your values into the *t* formula

- $t = \frac{M_D - 0}{s_{M_D}}$

- $\mu_D$  is calculated from null hypothesis, so  $\mu_D = 0$

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## Example

- A psychologist is interested in changing attitudes toward smoking
- Measures high school students' attitudes toward smoking (pre-test)
- Administers intervention (series of speakers, class discussions on consequences of smoking, poster campaign, etc.)
- Measures student attitudes toward smoking again (post-test)
- Was the anti-smoking intervention effective?

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## Example

Participant	Pretest (X <sub>1</sub> )	Posttest (X <sub>2</sub> )	D	D <sup>2</sup>
1	6	4		
2	7	4		
3	5	2		
4	5	5		
5	6	3		

\*scores are on 7 point scale, with higher scores indicating more favorable attitudes toward smoking

- 1) Hypotheses, 2) Critical Value of *t*, 3) Compute *t*, 4) Decision

Need to calculate  $M_D$ ,  $SS$ ,  $s^2$ ,  $s_{M_D}$ ,  $t$

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### Effect Size for Repeated *t*

- Cohen's  $d = M_D / s$ 
  - Just sample mean difference / sample standard deviation
- $r^2$  is the same as before
- Calculate for example - large effect?

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### Variability

- In repeated measures, variability describes consistency of effect
- High variability means that there is no consistent treatment effect
- High variability increases the size of the estimated standard error and reduces our *t* value
- The more variability there is, the less likely you are to obtain a significant treatment effect

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### One-tailed Repeated *t*

- Can use with directional hypotheses:
  - $H_0: \mu_D \geq 0$  or  $\mu_D \leq 0$
  - $H_1: \mu_D < 0$  or  $\mu_D > 0$
- Critical region is different
- Valence of *t* statistic matters (+ or -)
  - If  $M_D$  is neg, treatment decreased scores
  - If  $M_D$  is pos, treatment increased scores

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### Repeated vs. Independent Design

- Advantages:
  - Repeated measures typically requires fewer subjects
  - Repeated measures is best for looking at changes over time
  - Repeated measures reduces error (variance) caused by individual differences
    - even though the subjects in an independent-measures design are randomly assigned, it is possible that the two groups differ
    - because you are looking at difference scores, individual differences are eliminated (example)

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### Repeated vs. Independent Design

- Disadvantages:
  - Order effects (first treatment has effect on second)
  - Progressive error (function of time - practice effects or fatigue)
- But can correct for order effects by counterbalancing

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### Assumptions of Repeated $t$

- Observations within each treatment condition must be independent
- The population distribution of difference scores (D values) must be normal
  - Not a big concern unless sample is small

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### One more example:

- A researcher uses a repeated measures study to evaluate the effectiveness of magnetic therapy for treating chronic pain.
- **At Time 1:**  $n = 9$  patients are given a questionnaire that assess their current level of pain.
- **At Time 2:** after two weeks of magnetic therapy, their current level of pain is again assessed.
- For this sample, the reported level of pain decreased by an average of  $M_D = 12.5$  points with  $SS = 1152$ . Do these data indicate a significant change in the level of pain? Use a two tailed test with  $\alpha = .05$ .

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### APA format

- Include  $M$  and  $SD$  for both groups.
- Include statistic (and  $df$  for  $t$ )
- Include  $p$  value
- Indicate the direction of the effect if there is one:
  - As expected, Group A ( $M = xx, SD = xx$ ) scored significant higher than group B ( $M = xx, SD = xx$ ),  $t(df) = x.xx, p < .xx$ .
  - Contrary to predictions, men ( $M = xx, SD = xx$ ) were significantly slower to respond than women ( $M = xx, SD = xx$ ),  $t(df) = x.xx, p < .xx, d = .xx$ .
- etc.

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## Estimation

Chapter 12

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## Chap. 12 - Estimation

- Not everything in this chapter is important / explained well.
- It will be more important that you pay attention to your class notes than the book for this section.

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## Estimation

- The goal of **hypothesis testing** is to evaluate whether a treatment has any effect.
  - This is a yes or no question.
  - We set an alpha level, and if we reject the null we say "yes," but if we don't we say "no."
- The goal of **estimation** is to determine the value of the population mean after treatment.

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## Estimation

- **Point estimate:** Use a single number as your estimate of an unknown population parameter.
  - for example, how do you think we should estimate  $\mu$ ?
- **Confidence Intervals:** We find a range of values as an estimate for an unknown population parameter.
  - given a level of probability (e.g., 95%), compute a range of values for possible values of the population parameter
  - in the long run 95% of our computed confidence intervals will contain the population parameter
    - DOES NOT mean that there is a 95% chance that the population parameter lies within the computed interval.
    - Remember – probability in statistics applies to a long sequence of replication of an experiment (e.g., sampling distribution of means)

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## Confidence Interval

General formula:

- Confidence interval for population mean = sample mean +/- critical value of z or t \* standard error
- Lower bound: sample mean - critical value \* standard error
- Upper bound: sample mean + critical value \* standard error.

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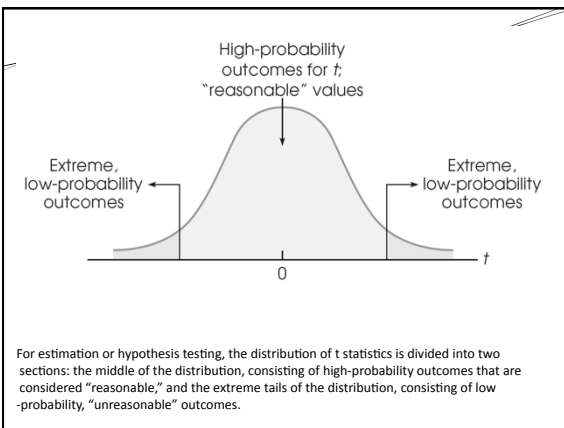
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## Specific formulas

- Single sample t-test
  - CI for  $\mu = M \pm t_{crit} * s_M$
- Independent samples t-test
  - $\mu_1 - \mu_2 = M_1 - M_2 \pm t_{crit} * s_{(M_1 - M_2)}$
- Dependent samples t-test
  - $\mu_D = M_D \pm t_{crit} * s_{MD}$
- You are estimating the population mean or mean difference, using sample data and the critical t-value of interest.

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### Single sample *t*-test

- $\mu = M \pm t * s_M$
- Example: A sample of 64 students from NJ scored a  $M = 83$  ( $SD = 12$ ) on a math test. Estimate the population mean score with 80% confidence.
- Need to figure out  $t$  and  $s_M$ 
  - 80% C.I., means 80% of the  $t$  distribution with  $df = n - 1 = 63$ .
    - What are the  $t$ -values we need?
  - How do we compute  $s_M$ 
    - $s_M = \sqrt{s^2/n}$

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### Computing CI for Independent *t*

- $\mu_1 - \mu_2 = (M_1 - M_2) \pm t_{S(M_1-M_2)}$
- Example - study habits (music vs. no noise)

Music group	No noise group
$n = 7$	$n = 9$
$M = 82$	$M = 90$
$SS = 100$	$SS = 125$

- First calculate pooled variance, and then standard error
- Next determine  $t$  boundaries

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### Computing CI for Repeated *t*

- Basically the same as with independent...
  - $\mu_D = M_D \pm t_{S_{M_D}}$
- Example - cholesterol lowering drug
  - $n = 15$ , measured at baseline, then after 6 weeks of drug treatment,  $M_D = 6$  points,  $SS = 1000$
- First calculate sample variance, and then standard error
- Next determine  $t$  boundaries

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