

<h2>Introduction to ANOVA</h2>	
	Chapter 13

<h3>Class Outline</h3>	
	<ul style="list-style-type: none"><input type="checkbox"/> Chap 13 – Intro to ANOVA<input type="checkbox"/> Questions on problem set

<h3>What type of t test would we use if:</h3>	
	<ul style="list-style-type: none"><input type="checkbox"/> A psychologist would like to study the effect of a new drug. A sample of people is selected and compared with a known population. <input type="checkbox"/> A psychologist would like to study the effect of a new drug. Two samples of people are selected; one is given the drug, one is not. The samples are then compared.

What type of t test would we use if:

- A psychologist would like to study the effect of a new drug. A sample of people is selected and measured both at baseline and after drug treatment. The measurements are then compared.
- A psychologist would like to study the effect of a new drug. Three samples of people are selected; one is given the drug, one is given a low dose, and one is given a high dose. The samples are then compared.
 - Can't use a t test because are 3 groups!

Intro to ANOVA – Chap. 13

- Analysis of variance (ANOVA) is a hypothesis testing procedure that compares mean differences between two OR MORE treatments (or populations)
- Major advantage over t tests:
 - t tests compare two populations at most
 - ANOVA compares two or more treatments

ANOVA versus several t -tests

- Why use ANOVA rather than several t -tests?
 - Practical reasons: testing for differences all in one test.
 - Power: increases if we test all at once.
 - Parsimony: one single result
 - Probability of Type I error reduced (not compounded)

Terminology

- In an analysis of variance, the variable (independent or quasi-independent) that designates the groups being compared is called a factor
- The individual conditions or groups that make up a factor are called the levels of the factor
 - Gender = 2 levels
 - No drug, high dose, low dose = 3 levels
- We will only be discussing single-factor, independent-measures today

Basic logic of the ANOVA

- **Within-groups variation:** the degree to which the individual scores vary about the sample mean *within each sample*.
- **Between-groups variation:** The degree to which the sample means vary *among themselves*.
- The researcher wants the between groups variability to be larger than the within groups variability.

Hypotheses

- Null hypothesis
 - $H_0: \mu_1 = \mu_2 = \mu_3 \dots = \mu_k$
- Alternative hypothesis
 - H_1 : at least one population mean is different from the others
 - (many different alternatives are possible – is too tedious to list them all)
 - H_1 : not H_0

The F-ratio

- Remember back to t tests:
 - $t = \frac{\text{obtained difference between sample means}}{\text{difference expected by chance}}$
- For ANOVA, test statistic is called an F-ratio
 - $F = \frac{\text{variance (differences) between sample means}}{\text{variance (differences) expected by chance (error)}}$
- Numerator measures the actual difference obtained from sample data and the denominator measures the difference that would be expected if there is no treatment effect (i.e., the degree to which the individual scores vary within each sample)

The F-ratio

- t and F are very similar, but:
- F is based on variance instead of sample mean difference like t test
 - (why it is called an analysis of variance)
- Cannot compute a sample mean difference if there are more than two means
- Example ($M_1 = 20$, $M_2 = 30$, $M_3 = 35$)

The F-ratio

- But, variance can tell us about differences between the groups

Set 1	Set 2
$M_1 = 20$	$M_1 = 28$
$M_2 = 30$	$M_2 = 30$
$M_3 = 35$	$M_3 = 31$

$$s^2 = 58.22$$

$$s^2 = 2.33$$

Logic of ANOVA

- In an ANOVA, we are analyzing variability
- We can get the total variance from all of our scores
 - Use the grand mean and all the scores from all the samples ($\sum(X - G)^2$)
- The total variability can be broken into two components
 - Between-treatments variance (differences between groups)
 - Within-treatment variance (inside each treatment group)

Between-Treatments Variance

- Measures how much difference exists between the treatment conditions
- Reflects chance variation *plus* any differential treatment effect (if one exists).
 - Treatment Effects – caused by treatment/ manipulation
 - Chance – differences not caused by treatment

Within-Treatments Variance

- Within each treatment condition, every individual is given the exact same treatment
- Yet even though each individual is given the same treatment, their scores will differ (vary)
- The within-treatments variance gives us a measure of how much difference we can expect by chance (when there is no treatment effect)

The F-ratio

- Now that we have divided the variability into two components, we compare them to determine the F-ratio

- $F = \frac{\text{variance between treatments}}{\text{variance within treatments}}$

The F-ratio

- $F = \frac{\text{treatment effect} + \text{differences due to chance}}{\text{differences due to chance}}$

- If treatment has no effect, the differences between treatments will be entirely due to chance. The numerator and the denominator in the F ratio are measuring chance differences and should be close to the same

- This would give us an F-ratio of close to 1.0 which would lead us to conclude that the treatment had no effect

The F-ratio

- If the treatment has an effect (difference between samples), then the between-treatment differences (numerator) should be larger than chance (denominator) and our F-ratio should be noticeably larger than 1.0

- Large F-ratio indicates that the differences between treatments are greater than chance or that our treatment has a significant effect

The F-ratio

- The denominator in an F-ratio is called the error term (measures only uncontrolled and unexplained variability)
- The numerator includes the same variability plus any differences caused by the treatment effect

Calculating ANOVA

- Because the calculations are going to get a bit more complicated, we need to introduce a new notation system for keeping track of everything
- Some of the new notation is familiar and some is new
- Note: there is no universally accepted system of notation for ANOVA

Notation

1. The letter k is used to identify the number of treatment conditions (number of levels of the factor)
2. The number of scores in each treatment is identified by n , yet the number of scores in the entire study is capital N – when all the samples are the same size, $N = kn$ (k multiplied by n)
3. The sum of the scores for each treatment ($\sum X$) is now identified by a capital T (treatment total)
4. The sum of all of the scores in the entire study (the grand total, $\sum T$) is identified by a capital G , and the grand mean is identified by \bar{X}

Formulas

- Next, we calculate the **Within-treatments Sum of Squares or SS_{within}**
- The variability inside each treatment condition
 - ▣ $SS_{\text{within}} = \sum SS_{\text{inside each treatment}}$
 - ▣ Calculate SS for each group using definitional ($\sum (X - M)^2$), or computational ($\sum X^2 - \frac{(\sum X)^2}{n}$) formula, then add together
 - ▣ Note that you are using each groups mean now, not the grand mean (i.e., each individual score in the group – group mean)².
 - ▣ If we had 3 groups, $SS_{\text{within}} = SS_1 + SS_2 + SS_3$

Formulas

- Last, we calculate the **Between-treatments Sum of Squares, or SS_{between}**
- The differences between treatment means
 - ▣ definitional formula for SS_{between} : $\sum (M - \bar{X})^2$
 - ▣ Step 1: Compute (sample mean – grand mean)² for each score
 - ▣ **Note:** This value will be the same for all scores in the group.
 - ▣ Step 2: Add together all of the values of $(M - \bar{X})^2$

Formulas

- computational formula for SS_{between} :

$$\sum \frac{T^2}{n} - \frac{G^2}{N}$$

- ▣ Step 1: Each treatment total is squared and then divided by the number of scores in the treatment. These scores are then added together.
- ▣ Step 2: The grand total is squared and divided by the total number of scores in the entire study.
- ▣ Step 3: The value of Step 2 is subtracted from the value of Step 1.

Formulas

- To check to make sure our calculations are correct

- ▣ $SS_{total} = SS_{between} + SS_{within}$

- This also means that if we have any two of the above, we can find the third

- Good idea to calculate using formulas first, though

Degrees of freedom

- We will be finding the total degrees of freedom, and then split it into $df_{between}$ and df_{within}

- ▣ $df_{total} = N - 1$

- ▣ $df_{between} = k - 1$

- ▣ $df_{within} = \sum df$ OR $N - k$

- Once again we can check our scores

- ▣ $df_{total} = df_{between} + df_{within}$

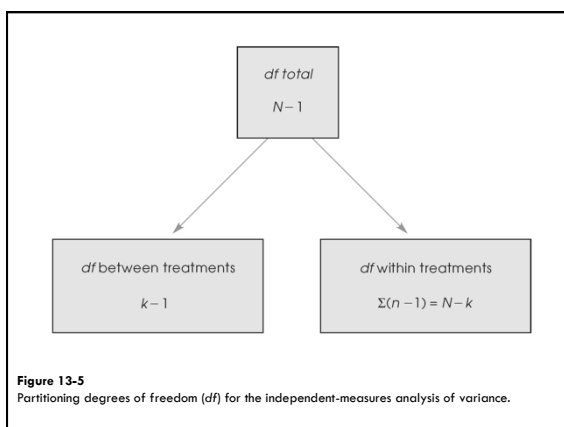


Figure 13-5
Partitioning degrees of freedom (df) for the independent-measures analysis of variance.

Remember...

- **Total** refers to entire set of scores (everybody who participated)
- **Within Treatments** refers to differences that exist inside the individual treatment conditions (compute SS and df inside each condition)
- **Between Treatments** refers to differences from one treatment to another (compare three means)

Mean Square

- Remember that the variance is the mean of the squared deviations
- In an ANOVA, instead of the word variance, we use MS (which stands for mean of the squared deviations)
- So for our F-ratio we will need to calculate the MS (variance) for our between treatments which will go in the numerator
- We then need to calculate the MS for our within treatments which will go in the denominator

Mean Square

- $MS_{\text{between}} = s^2_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}}$

- $MS_{\text{within}} = s^2_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}}$

- MS literally means variance ($s^2 = SS/df$)

F-ratio

□ Finally:

- $F = \frac{\text{variance between treatments}}{\text{variance within treatments}}$
- $F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$

ANOVA Table

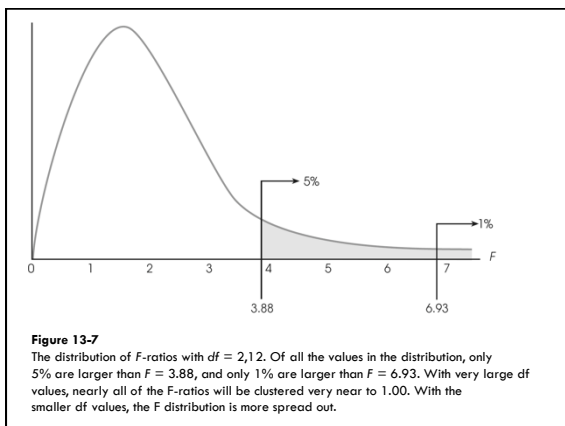
□ Everything should be put into an ANOVA table

Source	SS	df	MS	F
Between	--	--	--	--
Within	--	--	--	
Total	--	--		

- The two SS should add to the total SS
- The two df should add to the total df
- Fill in table as you work, so you don't get lost

Distribution of F-ratios

- Two characteristics of the F-distribution
 1. F values will always be positive numbers because we are using variance (which is always positive) to compute them
 2. When the null hypothesis is true, the F-ratio should be near 1.00. This means that the distribution of F-ratios should pile up around 1.00
- The exact shape of the distribution depends on the *df* (since *df* is used in calculating F statistic)



F distribution table (Appendix B, Table B.4)

- We need to know the df for the numerator as well as the denominator in order to find the critical value
- The df for the numerator are printed across the top and the df for the denominator are printed in a column on the left-hand side
- For a given study, we would write the degrees of freedom as:
 - $df = 2(\text{numerator}), 6(\text{denominator})$
 - $df = 2, 6$

Degrees of freedom: denominator	Degrees of freedom: numerator					
	1	2	3	4	5	6
10	4.96	4.10	3.71	3.48	3.33	3.22
	10.04	7.56	6.55	5.99	5.64	5.39
11	4.84	3.98	3.59	3.36	3.20	3.09
	9.65	7.20	6.22	5.67	5.32	5.07
12	4.75	3.88	3.49	3.26	3.11	3.00
	9.33	6.93	5.95	5.41	5.06	4.82
13	4.67	3.80	3.41	3.18	3.02	2.92
	9.07	6.70	5.74	5.20	4.86	4.62
14	4.60	3.74	3.34	3.11	2.96	2.85
	8.86	6.51	5.56	5.03	4.69	4.46

Table 13.3
 A portion of the F -distribution table. Entries in light type are critical values for the .05 level of significance, and bold type values are for the .01 level of significance. The critical values for $df = 2, 12$ have been highlighted (see text).

Example using computational formula

$$\square SS_{\text{TOTAL}} = \sum X^2 - \frac{G^2}{N}$$

$$\square SS_{\text{BETWEEN}} = \sum \frac{(T^2)}{n} - \frac{G^2}{N}$$

$$\square SS_{\text{WITHIN}} = \sum X^2 - \frac{(\sum X)^2}{n} \text{ (for each group)}$$
