TWO-FACTOR ANOVA

Chapter 14

CLASS OUTLINE – 8-5-08

- Two-factor ANOVA – Chap. 14
- ANOVA examples

TWO-FACTOR ANOVA

- So far, we've learned how to compute independent and repeated measures ANOVAs for one IV
  - Single-factor design
- But, most research uses more than one IV
  - More than one factor = factorial design
- Simplest version of a factorial design is a two-factor, independent measures, equal *n* design

TWO-FACTOR ANOVA

		Factor B: temperature		
		70° room	80° room	90° room
Factor A: humidity	Low humidity	Scores for n = 15 subjects tested in a 70° room with low humidity	Scores for n = 15 subjects tested in an 80° room with low humidity	Scores for n = 15 subjects tested in a 90° room with low humidity
	High humidity	Scores for n = 15 subjects tested in a 70° room with high humidity	Scores for n = 15 subjects tested in an 80° room with high humidity	Scores for n = 15 subjects tested in a 90° room with high humidity

TABLE 14-4
THE STRUCTURE OF A TWO-FACTOR EXPERIMENT PRESENTED AS A MATRIX. THE FACTORS ARE HUMIDITY AND TEMPERATURE. THERE ARE TWO LEVELS FOR THE HUMIDITY FACTOR (LOW AND HIGH), AND THERE ARE THREE LEVELS FOR THE TEMPERATURE FACTOR (70°, 80°, AND 90°).

TWO-FACTOR ANOVA

- We will be comparing three sets of mean differences:
  1. Mean differences between factor A (main effect)
  2. Mean differences between factor B (main effect)
  3. mean differences that may occur from the unique combination of factor A and factor B (interaction)
- We are combining three hypotheses into one analysis
- Each of these three hypotheses will be tested using a separate F-ratio

MAIN EFFECTS

- The mean differences among the levels of one factor are referred to as the main effect of that factor.
  - Sort of like testing a single-factor ANOVA
- Each main effect is evaluated with a hypothesis test to determine whether they are statistically significant
  - $F = \frac{\text{variance between the means for Factor A}}{\text{variance expected by chance/ error}}$

MAIN EFFECTS

- Hypotheses for factor A
  - H_0: $\mu_{A1} = \mu_{A2}$
  - H_1: $\mu_{A1} \neq \mu_{A2}$
- Hypotheses for factor B
  - H_0: $\mu_{B1} = \mu_{B2} = \mu_{B3 ...}$
  - H_1: H_0 is false

MAIN EFFECTS

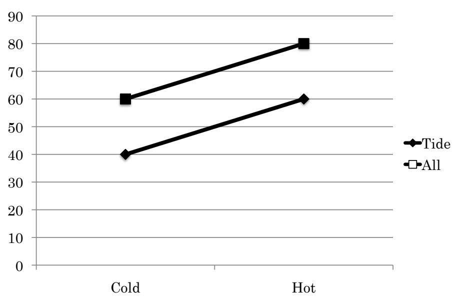
- Representing factors in a table can help you see where the main effects are
  - Rows vs. columns

	70°	80°	90°	
Low humidity	$M = 80$	$M = 80$	$M = 80$	$M = 80$
High humidity	$M = 80$	$M = 70$	$M = 60$	$M = 70$
	$M = 80$	$M = 75$	$M = 70$	

LAUNDRY DETERGENT EXAMPLE

	Tide	All	Marginal Mean
Cold	40%	60%	50%
Hot	60%	80%	70%
Marginal Mean	50%	70%	

Laundry Detergent Example



INTERACTIONS

- An **interaction between two factors** occurs whenever the mean differences between individual treatment conditions, or cells, are different from what would be predicted from the overall main effects of the factors
- Specific combinations of factor A and factor B may have different effects than either factor acting alone
- An interaction means that the effect of one factor depends on the other
  - (the two interact with one another to produce an effect that the two alone cannot explain)

INTERACTIONS

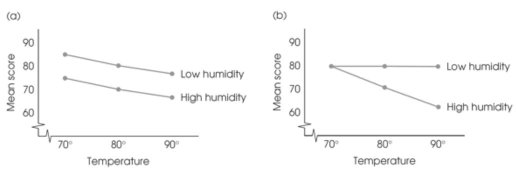
- Let's look at the data table again
  - Can you see a pattern in temperature that's different depending on humidity?

	70°	80°	90°	
Low humidity	$M = 80$	$M = 80$	$M = 80$	$M = 80$
High humidity	$M = 80$	$M = 70$	$M = 60$	$M = 70$
	$M = 80$	$M = 75$	$M = 70$	

INTERACTIONS

- $F = \dfrac{\text{variance not explained by main effects}}{\text{variance expected by chance/error}}$
- Hypotheses:
  - H_0: there is no interaction between factors A and B.
  - H_1: there is an interaction between factors

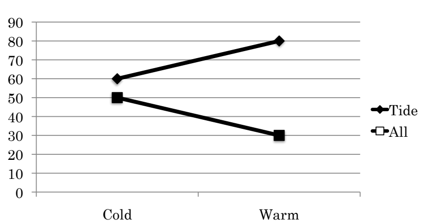
INTERACTIONS



No interaction — Interaction

Existence of non-parallel lines indicates an interaction between the two factors.

MAIN EFFECTS AND INTERACTIONS

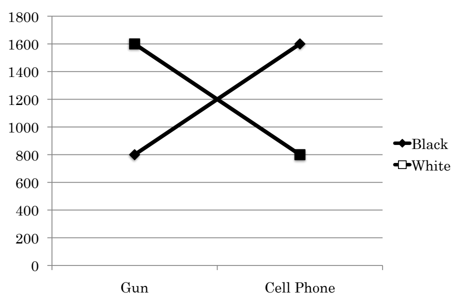


Does it seem like there is a main effect of temperature?
Does it seem like there is a main effect of detergent?
Does it seem like there is an interaction?

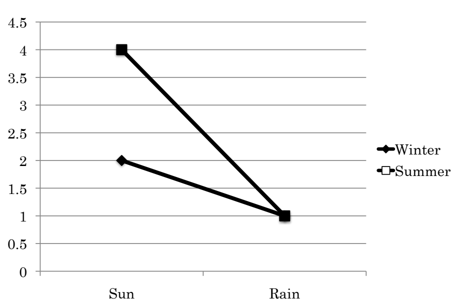
Main Effects and Interactions

	Cognitive-Behavioral Therapy	Rational Emotive Therapy	Marginal Means
Men	56	42	49
Women	36	34	35
Marginal Means	46	38	

Main Effects and Interactions



Main Effects and Interactions



COMPUTING A 2-FACTOR ANOVA

- We have 3 separate hypothesis tests in the two-factor ANOVA
  1. Is there a main effect of Factor A?
  2. Is there is main effect of Factor B?
  3. Is there an interaction?

COMPUTING A 2-FACTOR ANOVA

- The first stage of the ANOVA is exactly the same as before.
- We will separate the variability into between-treatments and within-treatments variability
- In the second stage, we will split the between-treatments variability into 3 components – differences due to factor A, differences due to factor B, and any remaining differences (the interaction)

COMPUTING A 2-FACTOR ANOVA

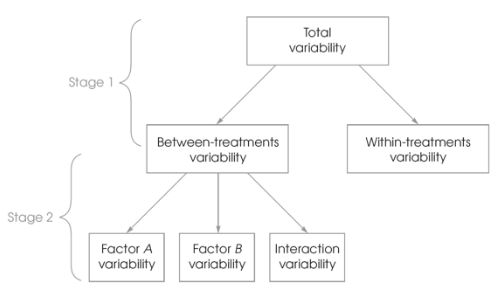


Figure 14-3
Structure of the analysis for a two-factor analysis of variance.

2-FACTOR SOURCE TABLE

Source	*SS*	*df*	*MS*	*F*
Between Treatments	--	--		
Factor A	--	--	--	--
Factor B	--	--	--	--
A x B	--	--	--	--
Within Treatments	--	--	--	
Total	--	--		

- When determining critical value, use $df_{between}$ for the specific factor and df_{within}

EXAMPLE

- Increasing the level of arousal/ motivation increases task performance
- But, with very difficult tasks, increased arousal may lead to decreased performance
- Does the effect of arousal on performance depend on task difficulty?

Factor *A*: Task difficulty		Factor *B*: Arousal level: Low	Medium	High	
	Easy	3 1 1 6 4 $T = 15$ $SS = 18$	2 5 9 7 7 $T = 30$ $SS = 28$	9 9 13 6 8 $T = 45$ $SS = 26$	$T_{ROW1} = 90$
	Difficult	0 2 0 0 3 $T = 5$ $SS = 8$	3 8 3 3 3 $T = 20$ $SS = 20$	0 0 0 5 0 $T = 5$ $SS = 20$	$T_{ROW2} = 30$
		$T_{COL1} = 20$	$T_{COL2} = 50$	$T_{COL3} = 50$	$N = 30$ $G = 120$ $\Sigma X^2 = 840$

TABLE 14-7
HYPOTHETICAL DATA FOR A TWO-FACTOR RESEARCH STUDY COMPARING TWO LEVELS OF TASK DIFFICULTY (EASY AND HARD) AND THREE LEVELS OF AROUSAL (LOW, MEDIUM, AND HIGH). THE STUDY INVOLVES A TOTAL OF SIX DIFFERENT TREATMENT CONDITIONS WITH $N = 5$ SUBJECTS IN EACH CONDITION.

Plan of Attack

- First, we find the total sum of squares.
- Then, we find the between treatments sum of squares, treating each cell as a separate treatment.
- We can also find the within treatments sum of squares (SS for each cell)

Plan of Attack

- Next, we find the sum of squares for Factor A (task difficulty)
  - Same as between treatments SS, only now we use row totals instead of cell totals
  - (collapsing across Factor B)
- We do the same for Factor B (arousal level)
  - Again, same as between treatments, only now we use column totals instead of cell totals
  - (collapsing across Factor A)
- Finally, the AxB interaction is simply:
  - $SS_{AxB} = SS_{between\ treatments} - SS_A - SS_B$

Plan of Attack

- Figuring out *df*
  - df_{TOTAL} is still N-1
  - df_{WITHIN} is the sum of the df for each cell
  - $df_{BETWEEN\ TREATMENTS}$ is # of cells – 1
    - df_A = # of levels of Factor A (rows) – 1
    - df_B = # of levels of Factor B (columns) – 1
    - $df_{AxB} = df_{BETWEEN\ TREATMENTS} - df_A - df_B$
- Compute MS
  - Same as before, just SS/*df*

Computing the F-Ratios

- Main effect of Factor A:

$$F_A = \frac{MS_A}{MS_{WITHIN}}$$

- Main effect of Factor B:

$$F_B = \frac{MS_B}{MS_{WITHIN}}$$

- Interaction effect:

$$F_{AxB} = \frac{MS_{AxB}}{MS_{WITHIN}}$$

INTERPRETING RESULTS

- Need to look at overall pattern of results
- Generally, we say that a significant interaction *qualifies* any significant main effects
- Can't take main effects at face value if there is a significant interaction

EFFECT SIZE

- Use η^2 (percentage of variability explained by treatment effects)
  - Compute one value for each *F* ratio calculated
  - Note – book has you compute partial η^2, I prefer regular η^2
- For Factor A: $\eta^2 = \frac{SS_A}{SS_{TOTAL}}$
- For Factor B: $\eta^2 = \frac{SS_B}{SS_{TOTAL}}$
- For AxB: $\eta^2 = \frac{SS_{AxB}}{SS_{TOTAL}}$

ASSUMPTIONS OF 2-FACTOR ANOVA

- Observations within each sample must be independent
- The populations from which the samples are selected must be normal (not very important if sample sizes are large)
- Homogeneity of variance

APA STYLE

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FIGURE 1

More Examples?