Z-scores and Standardized Distributions

CHAPTER 5

CLASS OUTLINE – 7-10-08

- Review
- Go over workshop #1
- Z-Scores, Standardized Distributions – Chap. 5
- Questions about problem set #1

CHAP. 5 – Z-SCORES

- We have learned about mean and standard deviation
- z-scores use the mean and the standard deviation of a distribution to standardize every score in the distribution
- The purpose of transforming individual scores (X values) into z-scores is to tell you exactly where each score lies in a distribution in relation to other scores

EXAMPLE

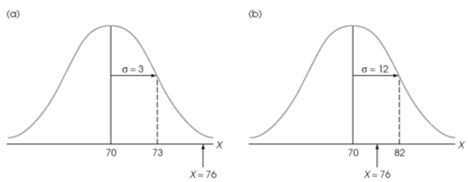


Figure 5.1 (p. 110)
Two distributions of exam scores. For both distributions, μ = 70, but for one distribution, σ = 12. The position of *X* = 76 is very different for these two distributions.

Z-SCORES

- Tell us:
  - 1) exact location of original X value within the distribution
  - 2) by standardizing scores, allow us to compare across multiple distributions
- For example, let's say you took the SAT and scored 580 on verbal
- When college is through, you want to go to graduate school and have to take the GRE. You score 590 on verbal.
- Did you do better on your SAT or GRE? By how much better or worse did you do? We will come back to this question later…

Z-SCORES

- You cannot tell anything about these scores until you standardize them.
- You cannot compare scores from 2 different distributions because the mean and standard deviation from each distribution will likely be different.
- **z-scores enable you to convert your raw scores into standardized scores that enable you to compare them to other standardized scores.**

LOCATION IN A DISTRIBUTION

- z-scores tell you exactly where a score lies in a distribution of scores.
- The + or – in front of the z-score tells you whether the score is located above or below the mean
- The number (z-score itself) tells you the distance of the score from the mean **in standard deviation units**

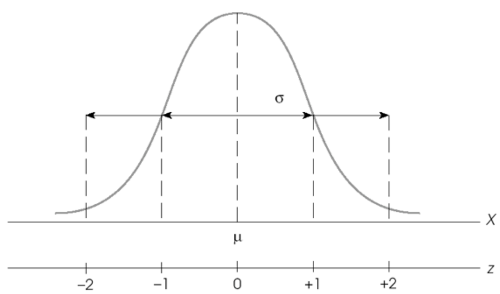


Figure 5-2 (p. 112)
The relationship between z-score values and locations in a population distribution.

LOCATION IN A DISTRIBUTION

- So for our SAT and GRE example, we need to know the mean and standard deviation before we can convert our scores into z-scores and compare them.
- If, for example, the mean for SAT verbal score was $\mu = 505$ with a standard deviation of $\sigma = 111$ and we had a score of 616, we would have a z-score of exactly $z = +1.00$ (our score of 616 is located exactly 1 standard deviation above the mean).
- I.Q. example – if have mean I.Q. of 100 and a standard deviation of 15, an I.Q. score of 70 has a z-score of what?

CALCULATING A Z-SCORE

- X values can be changed into z-scores just as z-scores can be changed into X values
- The formula for changing X values into z-scores is
  - $z = \frac{X - \mu}{\sigma}$
- X – μ is a deviation score (shows the distance between each score and the mean and whether the score is above or below the mean)
- We divide by the standard deviation because we want to measure the distance from the mean in standard deviation units

EXAMPLE

- You score an 85 on a test with mean 80, *SD* = 5
  - z = ?
  - You scored ______ above the mean
- You score an 89 on a test with mean 76, *SD* = 9
  - z = ?
  - You scored approx. ______ above the mean
  - More examples…

COMPUTING X FROM A Z-SCORE

- To transform z-scores into X values, we must manipulate the formula
- We first multiply the z-score by the standard deviation (zσ). Doing this shows us how far our score is from the mean.
  - Example
- We then add/subtract this score from the mean to get our X value:
  - $X = \mu + z\sigma$ (the + sign will be + or – depending on what $z\sigma$ equals)
- zσ is the deviation of X from the mean

COMPUTING X FROM A Z-SCORE

- So, if $z = 2.0$, $\mu = 10$, $\sigma = 5$
  - X = ?
  - X = ____________________

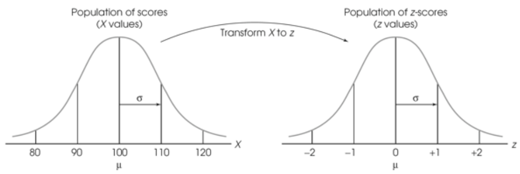
RELATIONSHIP OF Z, X, μ, AND σ

- If we know the mean, the z-score, and a particular X value, we are able to calculate the standard deviation.
  - $\mu = 66$, $X = 60$, $z = -2.0$
  - $\sigma = ?$
- If we know the standard deviation, the z-score, and a particular X value we are also able to calculate the mean.
  - $\sigma = 4$, $X = 40$, $z = +1.5$
  - $\mu = ?$

STANDARDIZING A DISTRIBUTION

- We are able to transform every raw score in our distribution into a distribution of z-scores
- This new distribution of z-scores will have 3 main properties:
  1. It will have the same shape as the distribution of X values (if the X distribution was normal, the z distribution will be normal)
  2. It will always have a mean of zero
  3. It will always have a standard deviation of one
- This z-score distribution is called a standardized distribution

STANDARDIZING A DISTRIBUTION



- Check to make sure standardized distribution has all 3 properties

STANDARDIZING AS RELABELING

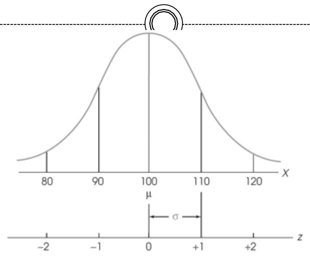


FIGURE 5-4
FOLLOWING A *Z*-SCORE TRANSFORMATION, THE *X*-AXIS IS RELABELED IN *Z*-SCORE UNITS.

COMPARING TWO DISTRIBUTIONS

- Can't compare apples to oranges
- Standardizing allows us to compare different scores from different distributions
- You take a test in 2 different classes
  - $X_1 = 9$, $\mu = 8$, $\sigma = 1$
  - $X_2 = 67$, $\mu = 60$, $\sigma = 5$
  - On which test did you perform better relative to the rest of the class?

OTHER STANDARDIZED DISTRIBUTIONS

- Z scores can be hard to understand because of + and – , and decimals
- Can create a new standardized distribution, with a specified mean and SD (whole round numbers)
  - Standardizing the distribution does not change an individuals place within the distribution (location of X relative to mean does not change)
- Process for standardizing:
  - Transform original scores into z-scores
  - Calculate the new standardized scores using these z-scores and your new mean and standard deviation

EXAMPLE

- Back to our SAT/ GRE example
- We can transform our SAT scores into z-scores with a mean of zero and a standard deviation of 1, but this is not what ETS actually does with SAT and GRE scores.
- Instead, we can create a standardized distribution based on a mean and standard deviation of our choosing (typically simpler and easier to understand)

EXAMPLE

- SAT Verbal = 580 (raw)
  - That year, $M = 505$, $SD = 111$
- GRE Verbal = 590 (raw)
  - That year, $M = 520$, $SD = 103$
- ETS converts the old mean (505) and old standard deviation (111) to 500 and 100, respectively.
- You are then able to convert each raw score into a z-score and finally into a standardized score based on the new mean and standard deviation

EXAMPLE

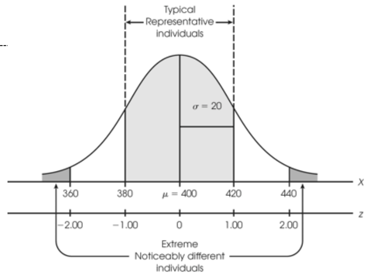
- raw SAT Verbal = 580
  - That year, M = 505, SD = 111
  - z =
  - New M = 500, SD = 100
  - Standardized score = z(SD) + M =
- raw GRE Verbal = 590
  - That year, M = 520, SD = 103
  - z = ?
  - New M = 500, SD = 100
  - Standardized score = ?
- On which test did you score higher?

Z-SCORES FOR SAMPLES

- Exact same formula as with populations, but use M and s instead of μ and σ
  - Remember to calculate s using $n - 1$
- $z = \frac{X - M}{s}$
- Distribution of sample z-scores will have same properties as distribution of population z-scores
  - $M_z = 0$, $s_z = 1.0$
- Only difference when working with a sample is that you calculate s using $n - 1$

USING Z-SCORES TO MAKE INFERENCES

- In experimental research, we have control group and treatment group
- Hypothesize that treatment group will be different than control
- Can use z-scores to test this
  - Did the treatment have an effect?



- If treatment zs = +/– 2.0, then it is reasonable to think that the treatment group is different than control group

Class Example

- Quiz scores:
  - Mike = 27
  - John = 24
  - Jenna = 19
  - Francesca = 30
  - Jamie = 22
  - Sam = 16
  - Joe = 28
  - Tom = 26
  - Dan = 6

- Calculate the *M* and *s*.
- Calculate each student's z-score.
- What do these z scores tell us?

Questions

ON TODAY'S MATERIAL
ON PROBLEM SET #1