

Introduction to the t statistic

Chapter 9

Class Outline – 7-22-08

- ▶ Collect problem set #2
- ▶ Review hypothesis testing
- ▶ Return Exam #1
- ▶ Intro to t statistic – Chap. 9
- ▶ New problem set (due on Thursday!)
- ▶ Questions

The t Statistic – Chap. 9

- ▶ In the last class, we used hypothesis testing to determine whether the mean of a random sample drawn from the population (and given a treatment of some kind), differed significantly from the population mean
- ▶ We did this by determining whether the mean of the sample was significantly less or greater than would be expected by chance
- ▶ However, there is a problem with using a z -score as an inferential statistic

The t statistic

- ▶ The problem is that to calculate a z-score, we must know the population standard deviation or variance in order to compute the standard error (yet we will rarely have this information)
- ▶ Often times, the whole reason we are conducting a hypothesis test is to learn about the unknown population parameters (this is the reason we have inferential statistics)

Example



The t statistic

- ▶ Because we will not normally know the population variance or standard deviation to calculate our standard error, **we must estimate the standard error**
- ▶ We do this by simply replacing σ^2 (population variance) with s^2 (sample variance) in our equation for standard error:
 - ▶ Standard error = $\sigma_M = \sqrt{\sigma^2 / n}$
 - ▶ Estimated standard error = $s_M = \sqrt{s^2 / n}$
 - Remember: $s^2 = SS / n - 1$

Estimated standard error (s_M)

- ▶ Used as an estimate of the real standard error σ_M when the value of σ^2 is unknown.
- ▶ It is computed from the sample variance (s^2) or sample standard deviation (s) and provides an estimate of the standard distance between a sample mean M and the population mean μ .

The t statistic

- ▶ Note that I am now using a slightly different formula for the standard error and estimated standard error
 - ▶ Now using the variance instead of the standard deviation
- ▶ This is not done to be annoying, but rather future formulas will require variance in the formulas for the estimated standard error and to keep things constant for future work, we are switching now
- ▶ The formulas, though they look different, are actually saying the exact same thing

The t statistic

- ▶ Standard error = $\sigma_M = \frac{\sigma}{\sqrt{n}}$ or $\sigma_M = \sqrt{\frac{\sigma^2}{n}}$
- ▶ Estimated standard error = $s_M = \frac{s}{\sqrt{n}}$ or $s_M = \sqrt{\frac{s^2}{n}}$
 - ▶ So, estimated standard error = $\sqrt{\text{sample variance} / \text{sample size}}$

The t statistic

- ▶ Now that we have our estimated standard error, we just put that in the place of the standard error to create the t statistic:

$$t = \frac{M - \mu}{s_M}$$

- ▶ The z-score formula is similar to the new t formula (we are only changing one thing from the z-score to the t formula)
- ▶ t-score approximates a z-score as n approaches infinity
- ▶ s^2 is a better representation of σ^2 as n approaches infinity

▶

The t Distribution

- ▶ If we have a z-score distribution computed from sample means (chap. 7), the distribution will tend to be normal
 - ▶ This is why we can consult the unit normal table, because we are assuming that the distribution is normal
- ▶ We can form a t distribution in the same way (distribution of sample means)
- ▶ However, how well a t distribution approximates a normal distribution is determined by degrees of freedom
 - ▶ There is a different sampling distribution of t for each possible number of df .

▶

Degrees of Freedom

- ▶ $df = n - 1$
- ▶ df = the number of scores in a sample that are free to vary. Because the sample mean places a restriction on the value of one score in the sample, there are $n - 1$ degrees of freedom for the sample (see page 96 for a review)

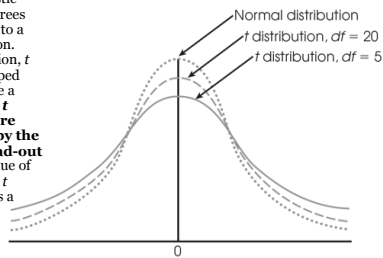
▶

The *t* Distribution

- ▶ A *t* distribution will more closely resemble a normal distribution as the degrees of freedom increase
- ▶ The larger the sample size, the larger the degrees of freedom, the better the *t* distribution approximates the normal distribution
- ▶ The *t* distribution has more variability than a *z* distribution, especially when the *df* values are small
 - ▶ adjusts for the introduction of variability in the denominator

The *t* Distribution

Figure 9-1 (p. 236)
 Distribution of the *t* statistic for different values of degrees of freedom are compared to a normal *z*-score distribution. Like the normal distribution, *t* distributions are bell-shaped and symmetrical and have a mean of zero. **However, *t* distributions have more variability, indicated by the flatter and more spread-out shape.** The larger the value of *df* is, the more closely the *t* distribution approximates a normal distribution.



The *t* Distribution

- ▶ Instead of using the unit normal table to look up the probability of obtaining a particular *z*-score, we must use a new table that takes the degrees of freedom into account
- ▶ The *t* distribution table (Appendix B, page 531) is used to look up the probability of obtaining a particular score

One-sample t test

- ▶ Step 1: State the Hypothesis
 - ▶ H_0 :
 - ▶ H_1 :
- ▶ Step 2: Locate the Critical Region
 - ▶ 2-tailed test, $\alpha = .05$
 - ▶ calculate df
 - ▶
 - ▶ look at t table
 - ▶ critical/ cut-off value for t
 - ▶

One-sample t test

- ▶ Step 3: Collect data, compute test statistic
 - ▶ A little more computation than with z score
 - ▶ Must calculate sample variance, estimated standard error, then t statistic
 - ▶ See calculations on board
- ▶ Step 4: Make a decision
 - ▶ Where does the obtained t fall within the t distribution?
 - ▶ If it is in the critical region, we can reject the null

Assumptions of t statistic

1. Sample must be independent observations
 - ▶ No predictable relationship between observations
2. Population that the sample was taken from must be normal
 - ▶ this assumption can be violated if the sample is large

Effect Size - d

- ▶ Can calculate Cohen's d for a t statistic
 - ▶ Cohen's $d = \frac{\text{mean difference}}{\text{sample standard deviation}}$
- ▶ In memory example:
 - ▶ Cohen's $d = \underline{\hspace{2cm}}$

Effect Size - r^2

- ▶ An alternative measure of effect size can be done by determining how much of the variability in your scores are explained by the treatment effect – **this is known as r^2**
- ▶ Treatment causes scores to vary (increase or decrease)
- ▶ How much of the variability in the scores obtained (sample) is due to the effect of the treatment

Effect Size - r^2

- ▶ $r^2 =$ percentage of variance accounted for by treatment
 - ▶ $\frac{\text{variability accounted for}}{\text{total variability}}$
- ▶ $r^2 = \frac{t^2}{t^2 + df}$
- ▶ .01 to .09 = small effect
- ▶ .09 to .25 = medium effect
- ▶ greater than .25 = large effect

Reporting Results

- ▶ At end of most chapters, is a section called "In the Literature"
- ▶ Pay attention to it!
- ▶ Example of reporting results on board.



More examples

New Problem Set
Questions??
